VIII-7. CHARACTERISTICS OF A PERIODIC TYPE OF MICROWAVE SAMPLING CAVITY

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Introduction. For years, various properties of gaseous media have been measured using microwave cavities as the sampling device. Although extensive work has been done in this field with respect to the design of refractometers and microwave spectrometers, very little effort has been directed toward the determination of the variation of sampling cavity characteristics caused by opening it to the sampled medium. In this paper, a special type of spaced-ring cavity will be analyzed and the limits to which it may be opened to the sampled medium examined in detail.

This particular type of microwave cavity was considered because it offers the possibility of increasing the ratio of open- to closed-cavity surface and at the same time reducing the temperature dependence of its resonant frequency. The fact that more open surface may be possible without seriously degrading the desirable characteristics of a cavity is most attractive when one considers the turbulence problem present when measuring the properties of gaseous media under flowthrough conditions.

Theoretical Analysis. A spaced-ring type of microwave sampling cavity is shown in Figure 1. Using the space harmonic approach along with several simplifying assumptions, the following condition equation is obtained:

$$\pi N_1 (k_o a) J_1 (k_o a) = \sum_{n \neq 0} \frac{1}{\tau_n a} \left[\frac{\sin (\beta_n \delta / L)}{\beta_n \delta / L} \right]^2 \qquad (1)$$

where for $n \neq 0$,

$$\tau_n a = \sqrt{(\beta_o + 2\pi n/L)^2 a^2 - k^2 a^2} \qquad (2)$$

and for $n = 0$,

$$\tau_o a \triangleq j k_o a = j \sqrt{k^2 a^2 - \beta_o^2 a^2} \qquad (3)$$

In these equations, $k = \omega \sqrt{\mu \varepsilon}$

It is now assumed that the foregoing derivation for an infinite waveguide periodic structure is also valid for a cavity made of the same elements. For the cavity case we require that:

$$\beta_o = P\pi/d \qquad P = 1, 2, 3 \ldots \qquad (4)$$

because of the end plates. The cavity modes of most interest are the TE_{01P} types because of their high Q_o values, where $k_o a = 3.835$ for the unperturbed or solid wall cylindrical cavity for all P. In order to solve for the actual $k_o a$ for a spaced-ring cavity structure, we use an iterative procedure. First, we let $k_o a = 3.835$ which is the value for the unperturbed cavity. Then one can calculate $\tau_n a$ using:

$$\tau_n a = \sqrt{4\pi^2 m^2 (a/d)^2 (nP/m + n^2) - k_o^2 a^2} \qquad (5)$$

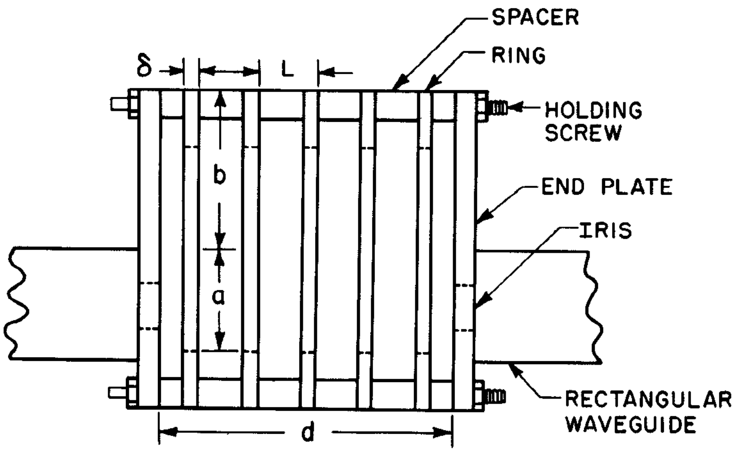

Figure 1. A Spaced-Ring Transmission-Type Sampling Cavity

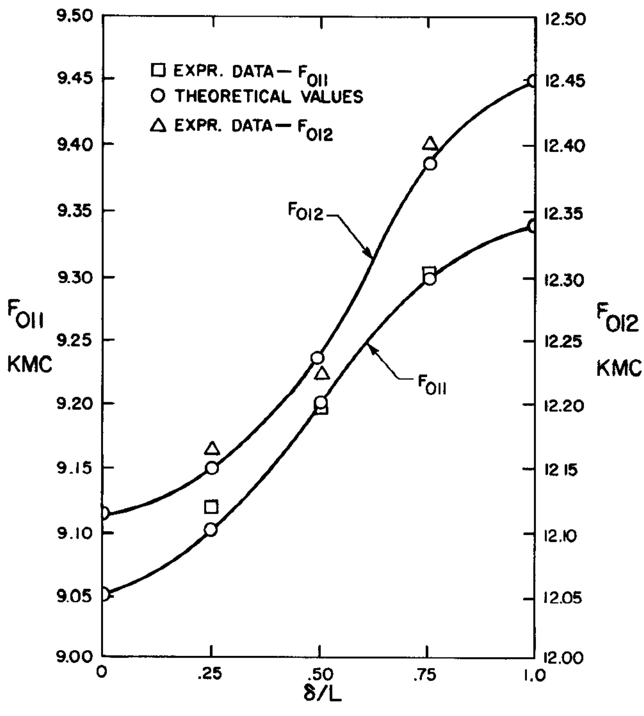

Figure 2. Cavity Resonant Frequency F_{011} and F_{012}

where P = number of half wavelengths in cavity of length d
n = order of space harmonic or index of summation
d = length of cavity
m = number of identical units of length L in total cavity length d.

One can now solve for a new $k_0 a$ value using eq. (1). Repeating this procedure in an iterative fashion, the value of $k_0 a$ can be found as accurately as desired. This value of $k_0 a$ can now be used to compute the resonant frequency for the cavity by using the equation:

$$F_{01P} = 1/2\pi a\sqrt{\mu\varepsilon}\sqrt{(k_0 a)^2 + (\pi P a/d)^2} \qquad (6)$$

Experimental and theoretical curves of F_{011} and F_{012} are plotted in Figure 2.

To determine the unloaded quality factor Q_0 for this type of cavity, one can use:

$$Q_0 = \frac{\pi^3 \varepsilon \mu^2 \omega^3 d a^4 \left[J_1^2(k_0 a) - J_0(k_0 a) J_2(k_0 a)\right]\left[J_1^2(k_0 a) + N_1^2(k_0 a)\right]}{\left\{\begin{array}{l} 2\pi^3 a^3 R_s k_0^2 \left[J_0^2(k_0 a)\{J_1^2(k_0 a) + N_1^2(k_0 a)\} + J_1^2(k_0 a)\{J_0^2(k_0 a) + N_0^2(k_0 a)\}\right] m\delta + \\ 4\pi^2 \omega\mu d a^2 J_1^2(k_0 a) + 4\pi^3 R_s \beta_0^2 a^4 \left[J_1^2(k_0 a) - J_0(k_0 a) J_2(k_0 a)\right]\left[J_1^2(k_0 a) + N_1^2(k_0 a)\right] \end{array}\right\}}$$

The experimental and theoretical values are plotted in Figures 3 and 4 for both the finite ring thickness $(b - a) > 0$ case and for the zero thickness $(b - a) = 0$ case. For the finite thickness case the radiation term in eq. (7) can be neglected. This is the second term in the denominator of this equation.

<u>Conclusions.</u> By decreasing the ratio of δ / L the turbulence caused by the rings in the flowing gas could be made quite small. This would tend to increase the frequency response and accuracy of the instrument using the cavity by decreasing the amount of mixing and the net wash-out time.

The material used for the spacers can be chosen so that the overall longitudinal temperature coefficient of expansion is smaller for a spaced-ring cavity than for the solid walled cavity. Thus it should be possible as the rings are made thinner to improve the stability of resonant frequency with changes in temperature.

As δ / L is made to approach zero for the finite wall thickness case, the characteristics of this type of cavity are improved in all of the factors previously mentioned. The approximation $J_1(k_0 a) \ll N_1(k_0 a)$ used in the derivation of eq. (1) has been shown to be valid in the cases considered for the TE_{011} and TE_{012} modes. For lower values of δ / L, $J_1(k_0 a)$ will become larger. This may cause the approximation made in order to solve the determinal equation to become somewhat less valid. This effect can be counteracted, however, by increasing m without lowering A-open/A-closed. The extension of this technique to larger values of L or smaller values of m is quite possible. In general, however, as L is increased for a given b - a dimension the radiation term will continue to increase and may not be negligible. The value of Q will be decreased if the radiation loss becomes significant compared to the wall losses. Also, this same type of effect would be present if b - a were reduced with other dimensions held constant. Although several terms were neglected in the final Q_0 equation, it turns out that each is quite small for the cases considered here compared to the terms retained.

This is also borne out by the close comparison of the theoretical values and experimental data as shown in Figs. 3 and 4 for the TE_{011} and TE_{012} modes. As the magnitude of the space harmonics outside the cavity approach those inside the cavity, the assumptions neglecting end effects outside the cavity may not be valid. This factor could be removed by extending the end plates in the +r direction for some reasonable length beyond $r = b$.

It has been shown conclusively that this type of cavity is well suited for the applications considered, and that it can be adequately described by the method of space harmonics for wide ranges in physical configuration.

References.

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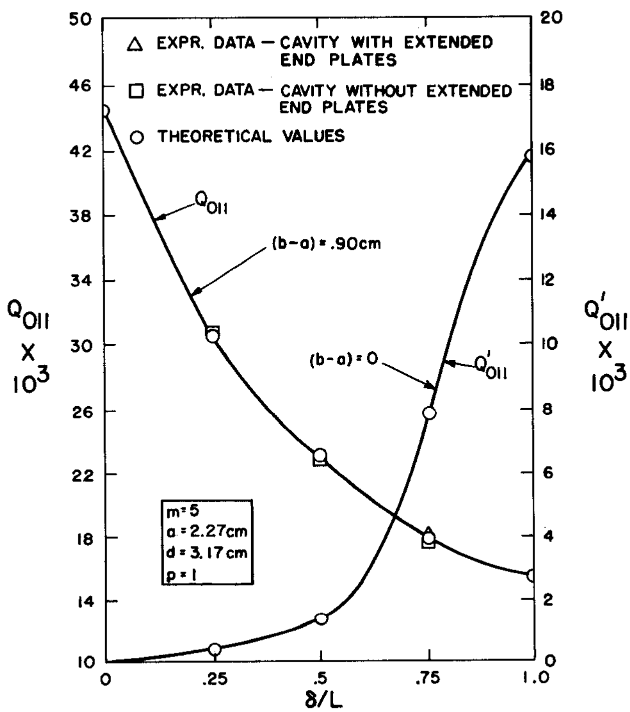

Figure 3. Theoretical and Measured Values of Q_{011} and Q_{011}

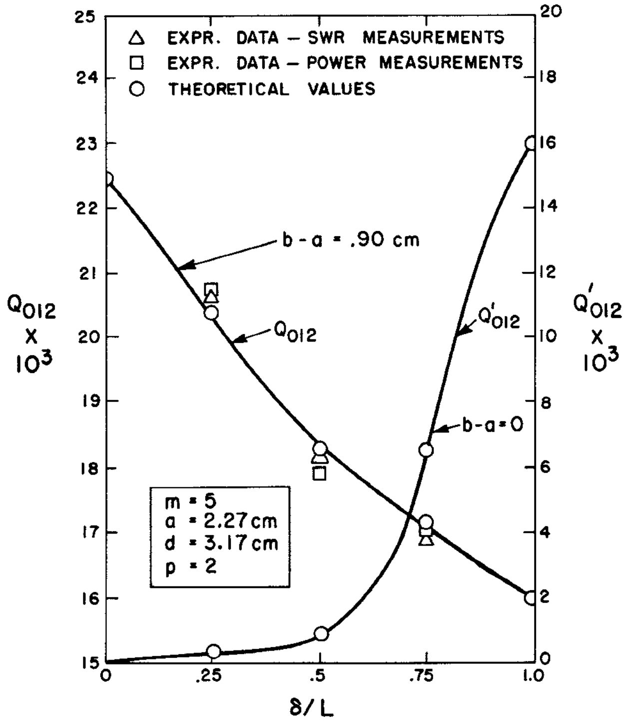

Figure 4. Theoretical and Measured Values of Q_{012} and Q_{012}'